

Exercise 19

Differentiate the function.

$$y = \ln(e^{-x} + xe^{-x})$$

Solution

Take the derivative of the function.

$$\begin{aligned}y' &= \frac{d}{dx} [\ln(e^{-x} + xe^{-x})] \\&= \frac{1}{e^{-x} + xe^{-x}} \cdot \frac{d}{dx}(e^{-x} + xe^{-x}) \\&= \frac{1}{e^{-x} + xe^{-x}} \cdot \left[\frac{d}{dx}(e^{-x}) + \frac{d}{dx}(xe^{-x}) \right] \\&= \frac{1}{e^{-x} + xe^{-x}} \cdot \left\{ \frac{d}{dx}(e^{-x}) + \left[\frac{d}{dx}(x) \right] e^{-x} + x \left[\frac{d}{dx}(e^{-x}) \right] \right\} \\&= \frac{1}{e^{-x} + xe^{-x}} \cdot \left\{ (e^{-x}) \cdot \frac{d}{dx}(-x) + (1)e^{-x} + x \left[(e^{-x}) \cdot \frac{d}{dx}(-x) \right] \right\} \\&= \frac{1}{e^{-x} + xe^{-x}} \cdot \{ (e^{-x}) \cdot (-1) + (1)e^{-x} + x [(e^{-x}) \cdot (-1)] \} \\&= \frac{1}{e^{-x} + xe^{-x}} \cdot (-e^{-x} + e^{-x} - xe^{-x}) \\&= \frac{-xe^{-x}}{e^{-x} + xe^{-x}} \cdot \frac{e^x}{e^x} \\&= -\frac{x}{1+x}\end{aligned}$$